Chapter 9: Session 23

Time Series Analysis: Introduction & Components of Time Series

A sequence of values which change with the course of time constitutes a Time Series.

Components of Time Series

A. Secular Trend

Secular trend is the general tendency of the data to grow, decline or to remain constant in values over a long period of time. It relates to the movement of data over a fairly long period of time.

There are two types of secular trends:

1. Linear Secular Trend

2. Non-Linear Secular Trend

Linear Secular trend is a straight line trend. When the data relating to a series is plotted against time, if most of the observations cluster around a straight line, it is a situation of linear trend. It can be upward slopping, downward slopping or be horizontal to the time axis. Model used for fitting Linear trend line is,

$$Yt = a + bt$$

Non-Linear Secular trend is a trend which does not give rise to a straight line when the time series is plotted against time. It takes a concave, convex or curvilinear form with ups and downs. Model used for fitting second degree trends is,

$$Yt = a + bt + ct^2$$

B. Cyclical Variation

Cyclical variation is the gradual fluctuation in a time series taking place over long time period (years). Business cycles present a common example of cyclical fluctuation, with a boom, slump, recession and recovery phases. Most of the time series relating to price, investment, income, wage, production, etc., exhibit this type of cycle.

The Residual Method is the common method used for calculating Cyclical variations. The ratio of actual values and the corresponding trend values is used as indicative of cyclical fluctuation.

Cyclical Variation, Ct = (Yt/ Y^t) * 100

where, Yt = Actual values

Y^t = Estimated trend values

C. Seasonal Variation

Seasonal Variation is fluctuations that occur regularly within a year over seasons. For instance, sale of refrigerator would be influenced by the seasons (summer, winter, autumn or rainy). These are short term fluctuations which can change weekly, monthly, quarterly or half yearly. The main reasons for such variations are natural causes such as weather or climate and social causes such as habits, customs, traditions, conventions and fashions.

A widely used technique for calculating the seasonal trends is the Ratio to Moving Average Method.

In general, moving average of a time series indicates running averages for the data taken over a given contiguous period. In the context of seasonal variation, we take the average over the number of periods in a year (4 if quarterly data, 12 if annual data). Each time the average is recorded at the centre of the period. If the number of periods is odd, then there is a unique centre. If it is even, then we centre the two middle most averages by taking their average, so as to represent against a particular period. It should be easy to see that these moving averages are smoothening out the seasonal effect. Consequently, the ratio of actual value to the corresponding moving average value would be indicative of the seasonal impact.

D. Irregular or Random Variation

Irregular variations follow an indistinct and an unequal pattern. They do not repeat in any specific pattern. They are also called erratic, accidental, episodic variations. These variations are caused by accidental and random factors like earthquakes, famines, floods, wars, strikes, lockouts, epidemics, etc. They include variations which are not attributable to secular, seasonal or cyclical variations.

There are no models to find out the irregular trend as they occur unexpectedly and inconsistently though some methods are used to isolate these trends.

Models used in Time Series

A time series can be expressed as:

Additive Model

Multiplicative model

In practice, the multiplicative model is popularly used. The multiplicative model is expressed as:

$Yt = Tt \times Ct \times St \times It$,

where,

Yt = Actual value of the time series at time t,

Tt = Trend value of the time series at time t.

Ct = Cyclical Index at time t

St = Seasonal Index at time t

It = Irregularity ratio at time t.

The purpose of studying a time series is to make forecasts for near future. Using multiplicative model, forecasting can be done taking into account the trend, cyclical and seasonal indices. We presume/expect the irregularity ratio to be unity on an average. Thus, a forecast based on multiplicative model would be more reliable.

Chapter 9: Session 24

Time Series: Moving Average, Fitting Linear and Second Degree Trends, Seasonal & Cyclical Variation

Secular Trend

Linear Secular trend is a straight line trend. When the data relating to a series is plotted against time, if most of the observations cluster around a straight line, it is a situation of linear trend. It can be upward slopping, downward slopping or be horizontal to the time axis.

Second Degree trend is a trend which does not give rise to a straight line when the time series is plotted against time. It takes a concave, convex or curvilinear form with ups and downs. One of the widely used method to fit a secular trend and estimate the model parameters is the Least Squares Method.

Commonly used Models for trend Fitting:

Linear Trend (Equations)

$$Yt = a + bt$$

Second Degree Trend

$$Yt = a + bt + ct^2$$

Least squares approach is used to fit the above trend curves. This approach takes care of linear trend and seasonality together.

Seasonal Variations:

Seasonal variation is repetitive and predictable movement around the trend line in one year or less. It is useful to project the past patterns into the future. Established seasonal patterns can eliminate its effects from the time series.

Cyclical Variation:

Cyclical variation is for longer periods than one year. The procedure that identifies cyclical variation is Residual method. There are two methods.

a) Cyclical variation as percentage of trend

$$= (y/\hat{y}) * 100$$

Where y = Actual Trend

$\hat{y}$ = Estimated Trend

b) Relative Cyclical Residual: It is the percent deviation from the trend is found in each value.

$$\{(y - \hat{y}) / \hat{y}\} * 100$$

Irregular Variation:

Irregular variation occurs over short intervals and follow random pattern.

Chapter 9: Session 25

Time Series: Problem Involving all components of Time Series

Problem Involving 4 components of a Time series:

Example of a problem using four components of Time Series:

Question:

The quarterly sales data (number of copies sold) for a college textbook over the years 2016, 2017 and 2018 are as follows.

Quarter	**2016**	**2017**	**2018**
1	1690	1800	1850
2	940	900	1100
3	2625	2900	2930
4	2500	2360	2615

A problem involving the four components of Time series has the following steps:

1. Deseasonalize the time series

Steps to Deseasonalize the Time series:

a. Find four-quarter moving average

b. Centre it by taking average of the two values at a time.

c. Deseasonalize by dividing the actual values with centred moving average

d. Compute the seasonal Index by taking quarterly values of every year, omitting the extreme values, and taking the average of the remaining values. These are called quarterly indices.

e. Add all quarterly indices

f. Calculate Adjusting Factor by dividing the Index with sum of modified mean

Adjusting Factor = Index / Sum of Modified Mean

	Computation of Moving Averages and Seasonal Index				
Year	Quarter	Actual Sales	4 quarter moving average	4 quarter centered moving average	Percentage of Acutal to Moving Average
2016	I	1690.00			
	II	940.00			
			1938.75		
	III	2625.00		1952.50	134.44
			1966.25		
	IV	2500.00		1961.25	127.47
			1956.25		
2017	I	1800.00		1990.63	90.42
			2025.00		
	II	900.00		2007.50	44.83
			1990.00		
	III	2900.00		1996.25	145.27
			2002.50		
	IV	2360.00		2027.50	116.40
			2052.50		
2018	I	1850.00		2056.25	89.97
			2060.00		
	II	1100.00		2091.88	52.58
			2123.75		
	III	2930.00			
	IV	2615.00			

	I	II	III	IV	
2016	--	--	134.443	127.4697	
2017	90.42386	44.83188	145.2724	116.3995	
2018	89.9696	52.5844	--	--	
Modified Sum	180.3935	97.41628	279.7154	243.8692	
Modified Mean	90.19673	48.70814	139.8577	121.9346	400.6972
Adjusting Factor	0.99826				

g. Multiply quarterly indices with adjusting factor to get seasonal indices

Quarter	Indices	Adjusting Factor	Seasonal Indices	Indices/100
I	90.20	0.998	90.04	0.90

II	48.71	0.998	48.62	0.49
III	139.86	0.998	139.61	1.40
IV	121.93	0.998	121.72	1.22

h. Calculate Deseasonalized Time Series values by dividing the actual values with Seasonal Index of quarter/100.

Year	**Quarter**	**Actual Sales**	**Seasonal Index / 100**	**Deseasonalized Index**
2016	I	1690.00	0.90	1876.95
	II	940.00	0.49	1933.23
	III	2625.00	1.40	1880.18
	IV	2500.00	1.22	2053.85
2017	I	1800.00	0.90	1999.12
	II	900.00	0.49	1850.96
	III	2900.00	1.40	2077.15
	IV	2360.00	1.22	1938.84
2018	I	1850.00	0.90	2054.65
	II	1100.00	0.49	2262.29
	III	2930.00	1.40	2098.64
	IV	2615.00	1.22	2148.33

2. Develop the Trend Line:

This is done by applying Least Square method to de-seasonlized time series. Steps to develop a Trend line are:

$$\hat{Y} = a + bx$$

a. Consider the deseasonalized values as ‘Y’ values.

b. Translate (or) code the time variable as ‘X’

c. Find XY, ΣXY

d. Find $X^{2,}$ ΣX^2

e. Find $\bar{y}$. Its value is same as ‘a’, the constant in the equation. Here it is found using the average of the Deseasonalized Index (Y)

f. Find ‘b’ using the formula b = $\Sigma XY / \Sigma X^2$

Year	Quarter	Deseasonalized Index (Y)	Coding x	xY	x-square
2016	I	1876.95	-11	-20646.43	121
	II	1933.23	-9	-17399.03	81
	III	1880.18	-7	-13161.25	49
	IV	2053.85	-5	-10269.26	25
2017	I	1999.12	-3	-5997.35	9
	II	1850.96	-1	-1850.96	1
	III	2077.15	1	2077.15	1
	IV	1938.84	3	5816.51	9
2018	I	2054.65	5	10273.24	25
	II	2262.29	7	15836.00	49
	III	2098.64	9	18887.74	81
	IV	2148.33	11	23631.63	121
		24174.17		7197.98	572

Y-bar = 2014.51

b= 12.58

a= 2014.51

g. Construct the equation:

$\hat{Y} = a + bx$ by substituting the values of ‘a’ and ‘b’ in the equation.

Trend Line = 2015+12.6X

h. Find the future quarter or year values using this equation.

3. Finding the cyclical variation around the trend line using Relative Cyclical Residual method.

$\{(y- \hat{y}) / \hat{y}\} *100$

Year	Quarter	Actual Sales (Y)	Coding x	ŷ=2015+12.6 X	Trend Line = (Y / ŷ)*100	Cyclical fluctuation around trend line {(y- ŷ) / ŷ } *100	(Y-ŷ)
2016	I	1690.00	-11	1876.4	90.07	-9.93	-186.4
	II	940.00	-9	1901.6	49.43	-50.57	-961.6

	III	2625.00	-7	1926.8	136.24	36.24	698.2
	IV	2500.00	-5	1952	128.07	28.07	548
2017	I	1800.00	-3	1977.2	91.04	-8.96	-177.2
	II	900.00	-1	2002.4	44.95	-55.05	-1102.4
	III	2900.00	1	2027.6	143.03	43.03	872.4
	IV	2360.00	3	2052.8	114.96	14.96	307.2
2018	I	1850.00	5	2078	89.03	-10.97	-228
	II	1100.00	7	2103.2	52.30	-47.70	-1003.2
	III	2930.00	9	2128.4	137.66	37.66	801.6
	IV	2615.00	11	2153.6	121.42	21.42	461.4

Chapter 10: Session 26

Decision Making – Introduction, Decision Making under Certainty & Uncertainty

Decision-making, both long-term and short-term, are an integral part of any management process. Decision theory helps us in arriving at appropriate decisions under different circumstances.

Courses of Action:

It is a list of available alternative acts available to the manager to address the problem. If n alternatives are available to problem, they are donated by A1…An.

States of Nature:

These are events which are beyond the control of the decision makers, but has an impact on the problem at hand and generally donated by S1…Sm to indicate m states of nature faced by the problem.

Payoffs:

This is a calculable measure of benefits or loss for each combination of action and state of nature. The payoff for i^{th} action and j^{th} state of nature is donated by X_{ij}.

Decision Making Environments:

1. Decision Making Under Certainty

Decision-making under certainty Under certain environment there is only one state of nature and all information are known with definite results..

2. Decision Making Under Uncertainty

Under uncertain environment, more than one states of nature exist. The types of decision making under uncertainty are

(a). Maximin Criterion: This is a pessimistic approach, where we go for the best action under the worst state of nature. Steps to consider for Maximin are:

- Choose the minimum payoffs of each alternative
- Choose the maximum of the minimums payoffs

(b). Minimax Criterion: This is an optimistic approach, where the worst pricing strategy for the best market conditions is selected. Here, this approach suggests the strategy of "substantial increase" in price with "no competition" as the best market condition.

- Choose the maximum payoffs of each alternative
- Choose the minimum of the maximums payoffs

(c). Maximax Criterion: Decision maker is aggressive by nature. This is the "best of the best" approach. Hence, the best market condition is "no competition" and the best strategy is "no increase in price". Steps to consider for Maximax are:

- Choose the maximum payoffs of each alternative
- Choose the maximum of the maximums payoffs

(d) Laplace Criterion: With no information, the probability of the various state of nature, we assume equal probability and compute the expected pay off for each action.

- Calculate the average payoff for each alternative
- Choose the alternative with highest average is the decision to be made

(e). Hurwitz Realism Criterion: The Maximax and Maximin are the two extremities - Optimistic and Pessimistic. Hurwitz proposed that realism would be somewhere in between. Representing the degree of optimism by œ (where $0 < \alpha < 1$), Hurwitz suggested that for each strategy (act) a decision index (Di) be calculated as the weighted average of optimistic and pessimistic pay offs, the former weighed by degree of optimism (α) and the latter by degree of pessimism ($1- \alpha$). Steps to consider for Hurwitz criterion are:

- Calculate Hurwitz value for each alternative
  =row maximum (α) + row minimum ($1- \alpha$)
- Choose the alternative with maximum Hurwitz value as the decision

(f) Regret criterion: This approach takes into account the loss of missed opportunity due to not knowing the state of nature in advance. An opportunity loss can be computed as the differences between the payoff for a given outcome and the maximum pay off under that state of nature. Steps to be followed for Regret Criterion are:

- Construct a 'Regret Table'
- Pick maximum regret of each row in regret table
- Pick minimum of the maximum. Its corresponding alternative is the decision

While there are differences in the conclusion reached following different criterion, they primarily reflect the underlying principles of each criteria. The company has to take a call on the principle to follow depending on its outlook and judgements.

Chapter 10: Session 27

Decision Making Under Risk

Decision Making under Risk

Decision Making under Risk considers Probability of occurrence. The three components of Decision making under Risk are:

(a). Expected Monetary Value (EMV)

In this case, we have only one criteria leading to unique selection of strategy, though probabilistic in nature. We find the expected pay off for each strategy called Expected Monetary Value (EMV), using the probabilities of the states of nature.

(b) Expected Opportunity Loss (EOL)

Under regret criterion, we discuss pay off in terms of opportunity loss. This approach would be followed if the payoffs were defined in terms of cost or downtime.

(c) Expected Value under Perfect Information (EVPI)

EVPI = EPPI – EMV(max)

Where, EPPI is Expected Profit with Perfect Information

Example:

Q: Center City Motor Sales has recently incorporated. Its chief asset is a franchise to sell automobiles of a major American manufacturer, CCMS' general manager is planning the staffing of the dealership's garage facilities. From information provided by the manufacturer and from other nearby dealerships, he has estimated the number of annual mechanic hours that the garage will be likely to need.

Hours	10,000	12,000	14,000	16,000
Probability	0.2	0.3	0.4	0.1

The manager plans to pay each mechanic $9.00 per hour and to charge customers $ 16. Mechanics will work a 40-hour week and get an annual 2-week vacation. Assume that mechanics are paid for their vacations.

i. Determine how many mechanics Center City should hire.

Ans. Number of work hours per year = 50 weeks * 40 hours

Each mechanic works 2000 hours/year

Calculate Expected Profit for each case

1. Expected Profit when 5 mechanics are hired.

Number of customers	Conditional Profit	Probability	Expected Profit = Conditional profit*probability
5	70,000	0.2	14000

6	70,000	0.3	21000
7	70,000	0.4	28000
8	70,000	0.1	7000
	Total Expected profit		**70000**

When 5 mechanics are hired,

- No. of mechanics hired = 5
- No. of hours/ year each mechanic works = 50 weeks * 40 hours = 2000 hrs
- No. of hours/ year 5 mechanics work = 5*2000 = 10000 hours
- Charge per hour = $16
- The profit earned by the company = 10,000 * 16 = $1,60,000
- The cost to the company = 10,000 hours * $9 = $90, 000
- The profit earned by the company for 5 mechanics = 1,60,000 – 90,000 = $70,000
  This is the conditional profit in the table.
- Expected profit = Σ (conditional profit * probability)
  = $70,000
- But, the company is also bearing the vacation costs of the 5 mechanics for 2 weeks = 5*2*40*9 = $3,600
- Final Expected profit when 5 mechanics are hired = 70,000 – 3600 =$ 66,400

2. Expected Profit when 6 mechanics are hired

- Cost per hour = $9
- Charge per hour/ customer = $16
- Profit per customer = $7
- Vacation cost of each mechanic for 2 weeks = 2*40hours *$9 = $720 per mechanic

a. Profit earned by the company by 6 mechanics per year = 6 *$7 *2000 hours = $84,000

- But, the company is also bearing the vacation costs of the 6 mechanics for 2 weeks = 6*2*40*9 = $4,320
- Total Expected Profit to the company if the company hires 6 mechanics and gets 6 or more customers = 84,000 – 4320 = $79680

b. If the company hires 6 mechanics but gets 5 customers

- It pays 1 mechanic who sits idle ($9 *2000) + 2week vacation of mechanic ($720) = $18,720
- Expected profit when 5 mechanics are hired = 70,000 – 3600 =$ 66,400
- Total Expected Profit to the company if the company hires 6 mechanics and gets only 5 customers = $66,400 – $18,720 = $47,680

a & b are the values used as conditional profit in the table.

3. Expected Profit when 7 mechanics are hired

- Cost per hour = $9
- Charge per hour/ customer = $16
- Profit per customer = $7
- Vacation cost of each mechanic for 2 weeks = 2*40hours *$9 = $720 per mechanic

a. Profit earned by the company by 7 mechanics per year = 7 *$7 *2000 hours = $98,000

- But, the company is also bearing the vacation costs of the 7 mechanics for 2 weeks = 7*2*40*9 = $5,040
- Total Expected Profit to the company if the company hires 7 mechanics and gets 7 or more customers = 98,000 – 5040 = $92960

b. If the company hires 7 mechanics but gets 5 customers

- It pays 2 mechanics who sits idle ($9 *2000 *2) + 2week vacation of mechanic ($720 *2) = $18,720 *2 = 37,440
- Expected profit when 5 mechanics are hired = 70,000 – 3600 =$ 66,400
- Total Expected Profit to the company if the company hires 7 mechanics and gets only 5 customers = $66,400 – $37,440 = $28,960

c. If the company hires 7 mechanics but gets 6 customers

- It pays 1 mechanic who sits idle ($9 *2000) + 2week vacation of mechanic ($720) = $18,720
- Expected Profit to the company if the company hires 6 mechanics = 84,000 – 4320 = $79680
- Total Expected Profit to the company if the company hires 7 mechanics and gets only 6 customers = $79680 – $18,720 = $60,960

a, b &c are the values used as conditional profit in the table.

4. Expected Profit when 8 mechanics are hired

- Cost per hour = $9
- Charge per hour/ customer = $16
- Profit per customer = $7
- Vacation cost of each mechanic for 2 weeks = 2*40hours *$9 = $720 per mechanic

a. Profit earned by the company by 8 mechanics per year = 8 *$7 *2000 hours = $112,000

- o But, the company is also bearing the vacation costs of the 8 mechanics for 2 weeks = 8*2*40*9 = $5,760
- o Total Expected Profit to the company if the company hires 8 mechanics and gets 8 or more customers = 112,000 – 5760 = $106,240

b. If the company hires 8 mechanics but gets 5 customers

- o It pays 3 mechanics who sits idle ($9 *2000 *2) + 2week vacation of mechanic ($720 *3) = $18,720 *3 = 56,160
- o Expected profit when 5 mechanics are hired = 70,000 – 3600 =$ 66,400
- o Total Expected Profit to the company if the company hires 8 mechanics and gets only 5 customers = $66,400 – $56,160 = $10,240

c. If the company hires 8 mechanics but gets 6 customers

- o It pays 2 mechanics who sits idle ($9 *2000 *2) + 2week vacation of mechanic ($720 *2) = $18,720 *2 = 37,440
- o Expected Profit to the company if the company hires 6 mechanics = 84,000 – 4320 = $79680
- o Total Expected Profit to the company if the company hires 8 mechanics and gets only 6 customers = $79,680 – $37,440 = $42,240

d. If the company hires 8 mechanics but gets 7 customers

- It pays 1 mechanic who sits idle ($9 *2000) + 2week vacation of mechanic ($720) = $18,720
- Expected Profit to the company if the company hires 7 mechanics = 98,000 – 5760 = $92,960
- Total Expected Profit to the company if the company hires 8 mechanics and gets only 7 customers = $92960 – $18,720 = $74,240

a, b, c &d are the values used as the conditional profits in the pay-off table.

Conditional Profits & expected Profits Payoff table:

	Mechanics Needed	5	6	7	8	Expected Profit
	Probability	0.2	0.3	0.4	0.1	
Mechanics Hired	5	66,400	66,400	66,400	66,400	66,400
	6	47,680	79,680	79,680	79,680	73,280
	7	28,960	60,960	92,960	92,960	70,560
	8	10,240	42,240	74,240	106,240	55,040

- Expected Profit = Σ (Conditional profit * probability)

Conclusion: Since the Expected profit is maximum ($73,280) when the company hires 6 mechanics, it should go for hiring 6 mechanics.

ii. How much should Center City pay to get perfect information about the number of mechanics it needs?

Solution:

It is found by calculating EVPI.

EVPI = EPPI – EMV

EPPI = 0.2 (66,400) + 0.3 (79,680) + 0.4 (92,960) + 0.1 (106240)

= 13,280+ 23904+37,184 + 10624

=84,992

EMV = Expected Profit = $73280

EVPI = 84,992 – 73,280 = $11,712

Chapter 10: Session 28

Decision Making – Decision Tree

Decision Tree is a diagrammatic presentation of the decision process, which helps essentially in easy comprehension of the logical relations in the process.

Decision Tree can be constructed using squares, circles and arrows

1. Squares: Decision node symbolizes decision points where a decision maker must choose among several possible actions. From decision node, one branch for each of the possible actions.

2. Circle: Chance node represents chance events, where some state of nature is realized. These chance of events are not under the control of decision maker. From chance nodes, one branch for each possible outcome.

Example:

The North Carolina Airport Authority is trying to solve a difficult problem with the over-crowded Raleigh-Durham airport. There are three options to consider.

1) The airport could be totally redesigned and rebuilt at a cost of \$8.2 million. The present value of increased revenue from a new airport is in question. There is a 70 percent probability this present value would be \$11.0 million, a 20 percent probability the present value would be \$5 million, and a 10 percent probability the present value would be \$1.0 million depending on whether the airport is a success, moderate success, or a failure.

2) The airport cold be remodelled with a new runway for a cost of \$4.7 million. The present value of increased revenue would be \$6.0 million (with probability of 0.8) or \$3.0 million (with probability 0.2)

3) They could do nothing with the airport and suffer a loss of revenue of either \$1 million (with probability 0.65) or \$4 million (with probability 0.2)

Questions for Discussion:

a. Construct a decision tree to help the Airport Authority.

Solution: The decision tree to help the Airport Authority is as follows:

b) Which option will maximize the present value of profit

i) Success of brand new airport = Total Revenue – Total Cost

\$8.8 million – \$8.2 million = \$0.6 million

Building a brand new airport would give a profit of \$0.6 million

ii) Remodel the airport with new runway = Total Revenue – Total Cost

\$5.4 million – \$4.7 million = \$0.7 million

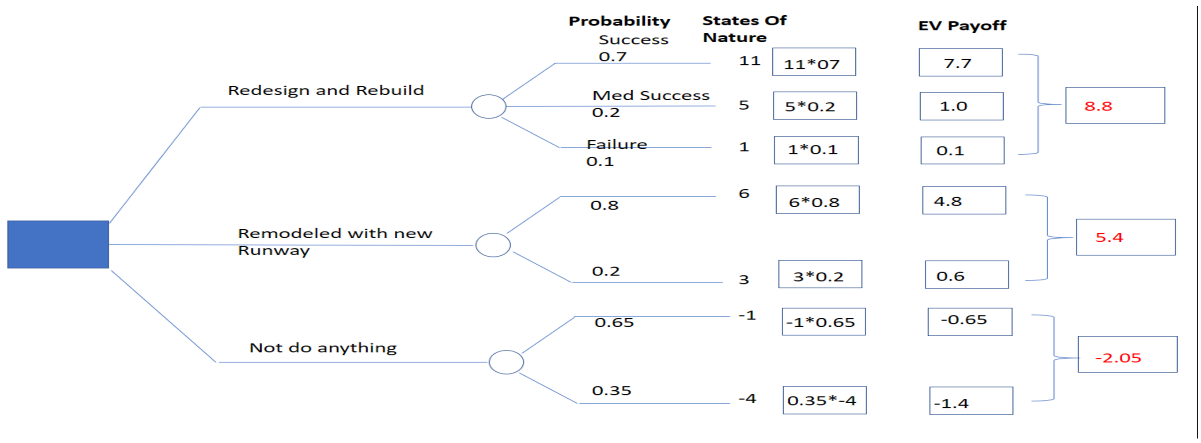
Remodelling the airport with a new runway would give a profit of \$0.7 million

iii) Not do anything = Total Cost

– \$2.05 million

Not doing anything would give them an additional cost of \$2.05 million

Since remodelling the airport with a new runway is giving the maximum profit, North Carolina Airport Authority should go for Remodelling the airport with a new runway.



c) How much would we be willing to pay for perfect information about success of a brand new airport?

With perfect Information about success of a brand new airport, we would invest in it if we knew it would be successful but would invest in Runway otherwise. Hence, the Expected Value with Perfect Information (EVPI) would be

EVPI = EPPI – EMV(max) Where,

EPPI is the Expected Profit with Perfect Information = 0.7 (2.8) + 0.3 (0.7) = 2.17

EMV is the Expected Monetary Value for brand new airport = 0.7

Therefore, EVPI = 2.17 – 0.7 = 1.47

Expected value with Perfect Information for brand new airport is $1.47million.

d) How much would we be willing to pay for perfect information about success of a brand remodelled airport?

With perfect Information about success of a brand new airport, we would invest in it if we knew it would be successful but would invest in Runway otherwise. Hence, the Expected Value with Perfect Information (EVPI) would be

EVPI = EPPI – EMV(max)

Where,

EPPI is the Expected Profit with Perfect Information = 0.8 (1.3) + 0.2 (0.6) = 1.16

EMV is the Expected Monetary Value for remodelling is = 0.7

Therefore, EVPI = 1.16 – 0.7 = 0.46

Expected value with Perfect Information for remodelling the airport is $0.46 million.

Chapter 11: Session 29

Linear Programming Problem

LP model is extensively being used in all functional areas of management, airlines, agriculture, military operations, oil refining, education, energy planning, pollution control, transportation planning and scheduling, research and development, health care systems, etc

Decision Variables

Represent the entities about which a decision is to be taken. They consume resources. Denoted by $x1, x2, x3, \ldots, xn$. In an LP model all decision variables are continuous, controllable and non-negative – $x1 \geq 0, x2 \geq 0, x3 \geq 0, \ldots, xn \geq 0$

Objective Function

The goal function of each LP problem, expressed in terms of decision variables to optimize the criterion of optimality i.e., measure-of-performance.

e.g. Maximization of profit or revenue

e.g. Minimization of cost or time or distance.

Objective Function is represented as

Max $Z = c1x1 + c2x2 + c3x3 + \ldots + cnxn$ or

Min $Z = c1x1 + c2x2 + c3x3 + \ldots + cnxn$

Where Z is the measure-of-performance variable and is a function of $x1, x2, \ldots, xn$

Quantities $c1, c2, c3, \ldots, cn$ represent the contribution of a unit of respective variable $x1, x2, x3, \ldots, xn$

Constraints

Constraints represent the limitations on the availability or use of resources. Limit the degree to which an objective can be achieved. Solution to the LP model must satisfy these constraints. Constraints are expressed as linear equalities / inequalities in terms of the decision variables.e.g. labour, machine, raw material, space, money.

Non-negativity Constraint

Non-negativity constraint is a restriction imposed on the decision variables. It ensures negative values are not assigned to the decision variables and are expressed as $x1, x2, x3, \ldots, xn \geq 0$

After an LPP is formulated the solution can be obtained using the following methods.

- Graphical Method Restricted to an LPP of two decision variables
- Simplex Method No restriction on the number of decision variables

Chapter 11: Session 30

LPP Graphical Method – Maximization Problem

Example: Following example uses the application of concepts and steps of LPP.

Q. A manufacturer produces the types of models M1 and M2. Each M1 model requires 4 hours of grinding and 2 hours of polishing. Whereas each M2 model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinders works for 40 hours a week and each polisher's works for 60 hours a week. Profit on M1 model is 3/- and M2 model is 4/-. Formulate the L.P.P to the above situation.

a. Construction of the Model:

1. Identify the objective of the case under discussion:

Since the profits on each model are mentioned, the objective of the case is to 'maximize the profits' for the two models, M1 and M2.

2. Define the constraints for achieving the objective

Constraints are the limitations within which the organization has to function as the resources are scarce and limited. Therefore, the three types of constraints mentioned are

a. Grinding Constraint

b. Polishing Constraint

c. Time Constraint

3. Define the Decision Variables to be considered

a. The number of units of Grinders let's say A

b. The number of units of Polishers let's say B

4. Write the objective in terms of these decision variables

Total Profit contribution = $3A + 4B$ which is written as, maximizing the function Z,

Max $Z = 3A + 4B$

5. List out the constraints in terms of decision variables

The objective function is subject to the following constraints

$4A + 2B \leq 40$ (Grinding hour constraint)

$2A+5B \leq 60$ (Polishing hour constraint)

6. State the Non-Negativity Constraints

$A, B \geq 0$

Solving the Problem:

The problem is solved using graphical method. The constraint equations are considered for drawing a graph.

i) Consider the constraint equations. Change the inequality into equation.

a. $4A + 2B \leq 40$ (Grinding hour constraint)

$4A + 2B = 40$

b. $2A+5B \leq 60$ (Polishing hour constraint)

$2A+5B =60$

ii) Find the coordinates of X-axis and Y-axis, from the equations

a) Coordinates are found by taking one variable as zero. A values are found by substituting B=0. And B values are found by substituting A = 0

4A + 2B = 40

Keeping B as zero, find A

4A + 0 =40

A = 10 when B =0 (10,0)

Next, by keeping A as zero find B

0+2B = 40

B = 20 when A = 0 (0,20)

So the coordinates to draw on the graph for the equation 4A + 2B ≤ 40 (Grinding hour constraint), are (10, 0) on X-axis and (0,20) on Y-axis. Since, the equation has '≤' sign, shade the area below the slope that is obtained by taking the above mentioned coordinates.

b) Coordinates are found by taking one variable as zero. A values are found by substituting B=0. And B values are found by substituting A = 0

2A+5B =60

Keeping B as zero, find A

2A + 0 =60

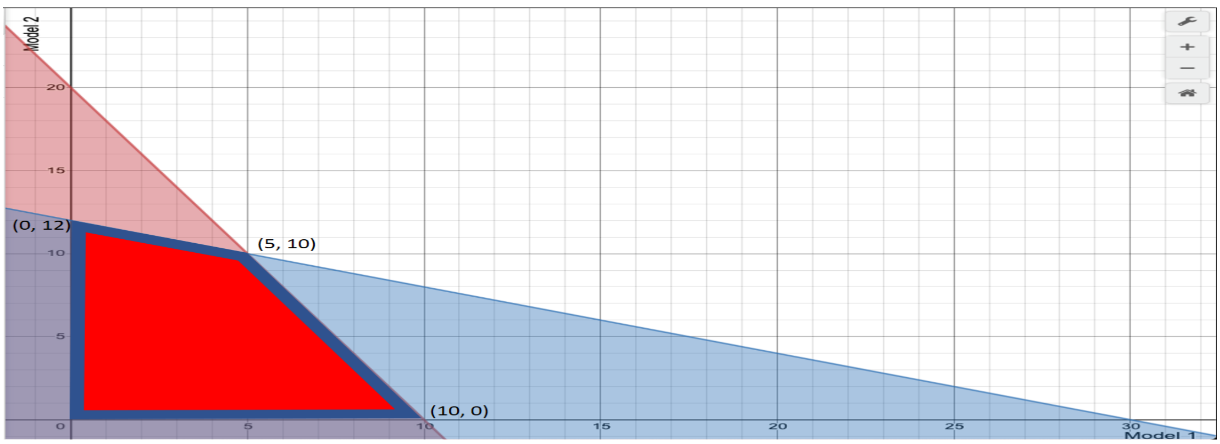
A = 30 when B =0 (30,0)

Next, by keeping A as zero find B

0+5B = 60

B = 12 when A = 0 (0,12)

iii) Identify the common area to both the equations, which is called the feasible region. Find the extreme coordinates of this common area. These extreme coordinates of the feasible region generally provide the optimized solution. Maximum values in case of Maximization of profits and minimum values in case of cost minimization. Therefore, the coordinates (0,12), (5, 10) and (10,0) are the extreme coordinates that can provide the maximum profits for Model 1 and Model 2.



iv) Find the optimized solution by substituting the identified coordinate values in the 'objective function' mentioned.

The objective function is Max Z = 3A + 4B

Objective Function	Co-ordinates of A and B	Z value = 3A + 4B	Z =	Profit Maximization Z=
Z= 3A + 4B	0,12	(3*0) + (4*12)	48 units	55 units.
	5,10	(3*5) + (4*10)	55 units	
	10,0	(3*10) + (4*0)	30 units	

Since it is a maximization of profits case, the coordinates that maximizes 'Z' function, should be considered as the number of units of A and B respectively, that gives maximum profits. Therefore, with the existing grinding and polishing constraints, 5 units of model M1 and 10 units of model M2 per hour gives maximum profits to the organization.

Chapter 11: Session 31

LPP Graphical Method – Minimization Problem

Vitamins V and W are found in two different foods F1 and F2.One unit of food F1 contains 2 units of vitamin V and 5 units of vitamin W. One unit of food F2 contains 4 units of vitamin V and 2 units of vitamin W. One unit of food F1 and F2 costs Rs. 30 and 25 respectively. The minimum daily requirements (for a person) of vitamin V and W is 40 and 50 units respectively. Assuming that anything in excess of daily minimum requirement of vitamin V and W is not harmful, find out the optimal mixture of food F1 and F2 at the minimum cost which meets the daily minimum requirement of vitamins V and w. Formulate this LPP

a. Construction of the Model:

1. Since the costs on each food are mentioned, the objective of the case is to 'minimize the costs' for the two foods, F1 and F2.

2. Constraints are the limitations within which the organization has to function as the resources are scarce and limited. Therefore, the two types of constraints mentioned are

    a. Vitamin V Constraint

    b. Vitamin W Constraint

3. The number of units of food F1 let's say A. The number of units of foodF2 let's say B

4. Total cost = 30A + 25B which is written as, minimizing the function Z,

    Min Z = 30A + 25B

5. The objective function is subject to the following constraints

    $2A + 4B \geq 40$ (Vitamin V constraint)

    $5A+2B \geq 50$ (Vitamin W constraint)

6. $A, B \geq 0$

Solving the Problem:

The problem is solved using graphical method. The constraint equations are considered for drawing a graph.

i) a. $2A + 4B \geq 40$ (Vitamin V constraint)

$2A + 4B = 40$

b. $5A+2B \geq 50$ (Vitamin W constraint)

$5A+2B =50$

ii) a) Coordinates are found by taking one variable as zero. A values are found by substituting B=0. And B values are found by substituting A = 0. For the equation, **2A + 4B = 40**

Keeping B as zero, find A

$2A + 0 =40$

A = 20 when B =0 (20,0)

Next, by keeping A as zero find B

$0+4B = 40$

B = 10 when A = 0 (0,10)

So the coordinates to draw on the graph for the equation $2A + 4B \geq 40$ (Vitamin V constraint) are (20, 0) on X-axis and (0,10) on Y-axis.

b) Coordinates are found by taking one variable as zero. ‘A’ values are found by substituting B=0. And B values are found by substituting A = 0. For the equation, **5A+2B =50**

Keeping B as zero, find A

$5A+2B =50$

A = 10 when B =0 (10,0)

Next, by keeping A as zero find B

$0+2B = 50$

B = 25 when A = 0 (0,25)

So the coordinates to draw on the graph for the equation $5A+2B \geq 50$ (Vitamin W constraint) are (10, 0) on X-axis and (0,25) on Y-axis.

iii) Maximum values in case of Maximization of profits and minimum values in case of cost minimization. Therefore, the coordinates (0,25), (7.5, 6.25) and (20,0) are the extreme coordinates that can provide the minimum costs for food 1 and food 2.

iv) Find the optimized solution by substituting the identified coordinate values in the ‘objective function’ mentioned.

The objective function is Min Z = 30A + 25B

Objective Function	Co-ordinates of A and B	Z value = 30A + 25B	Z =	cost Minimization Z=
Z = 30A + 25B	0,25	(30*0) + (25*25)	625	381.25 is the minimum cost
	7.5,6.25	(30*7.5) + (25*6.25)	381.25	
	20,0	(30*20) + (25*0)	600	

Since it is a minimization of costs case, the coordinates that minimizes 'Z' function, provides the optimal mixture of F1 and F2 respectively, that gives minimum profits. Therefore, with the existing Vitamin V and Vitamin W constraints, 7.5 units of F1 and 6.25 units of F2 provides minimum costs to the organization.

Chapter 11: Session 32

Formulation of a Transportation & Assignment Problem

Network flow problems are a special branch of LPP comprising of the following:

a. Transportation Problem (One-to-many)

A transportation problem is a special minimization problem that helps in minimizing the transportation costs to mobilize the vehicles from one place to many places.

b. Assignment Problem (One-to-one)

An Assignment problem constructs a model to assign one job to one work station in such a way that minimizes the costs or maximizes the profits.

After a Transportation Problem / Assignment Problem has been formulated as an LPP, the problem can be solved by simplex method.

LPP Formulation of a Transportation Problem:

The transportation problem seeks to minimize the total shipping costs of transporting goods from i sources (each with a supply si) to j destinations (each with a demand dj), when the unit shipping cost from source, i, to a destination, j, is cij**.** The steps followed are as follows:

1. Objective Function:

The linear programming formulation in terms of the amounts shipped from the sources to the destinations, xij, can be written as objective Function.

Minimize total transportation cost

Min Z = ∑ j=1 n ∑ i=1 m (cij∗xij)

2. Subject to constraints

∑ i=1 m (cij∗xij) ≤ si for each source i (supply constraints)

∑ j=1 n (cij∗xji) ≥ dj for each destination j (destination constraints)

3. Non-negativity constraints : xij, xji ≥ 0

Example:

Q. A product is produced at three plants and shipped to three warehouses (the transportation costs per unit are as follows)

Warehouse				
Plant	W1	W2	W3	Plant Capacity
P1	20	16	24	300
P2	10	10	8	500
P3	12	18	10	100
Warehouse Demand	200	400	300	

Develop a Linear Programming Model for minimizing the transportation costs.

Decision Variables

x_{11} = No of units transported from P1 to W1

x_{12} = No of units transported from P1 to W2

x_{13} = No of units transported from P1to W3

x_{21}= No of units transported from P2 to W1

x_{22} = No of units transported from P2 to W2

x_{23} = No of units transported from P2 to W3

x_{31} = No of units transported from P3 to W1

x_{32} = No of units transported from P3 to W2

x_{31}= No of units transported from P3 to W3

1. Objective Function Min Z = $20\ x_{11} + 16\ x_{12} + 24\ x_{13} + 10\ x_{21} + 10\ x_{22} + 8\ x_{23} + 12\ x_{31} + 18\ x_{32} + 10\ x_{33}$

2. Subject to constraints

Supply constraints

$x_{11} + x_{12} + x_{13} \leq 300$

$x_{21} + x_{22} + x_{23} \leq 500$

$x_{31} + x_{32} + x_{33} \leq 100$

Demand Constraints

$x_{11} + x_{12} + x_{13} \leq 200$

$x_{21} + x_{22} + x_{23} \leq 400$

$x_{31} + x_{32} + x_{33} \leq 300$

3. Non-negativity Constraints : $x_{11}, x_{12}, x_{13} \ldots\ldots x_{32}, x_{33} \geq 0$

LPP Formulation of an Assignment Problem:

An assignment problem seeks to minimize the total cost assignment of m workers to m jobs, given that the cost of worker i performing job j is cij. It works on the assumption that all workers are assigned and each job is performed

An assignment problem is a special case of a transportation problem in which all supplies and all demands are equal to 1. The linear programming formulation of an Assignment problem includes the following steps:

1. Objective Function:

Min $Z = \sum\sum (cij * xij)$

2. Subject to constraints

$\sum_{j=1} (xij) = 1$

$\sum_{i=1} (xij) = 1$

3. Non-Negativity Constraint

$xij \geq 0$

Example:

Q.Scott and Associates, Inc., is an accounting firm that has three new clients. Project leaders will be assigned to the three clients. Based on the different backgrounds and experiences of the leaders, the various leader-client assignments differ in terms of projected completion times. The possible assignments and the estimated completion times in days are as follows:

Client			
Project Leader	X	Y	Z
A	10	16	32
B	14	22	40
C	22	24	34

Formulate the problem as a linear program.

Ans. Decision Variables

x_{11} = Time of PL A on client X

x_{12} = Time of PL A on client Y

x_{13} = Time of PL A on client Z

x_{21} = Time of PL B on client X

x_{22} = Time of PL B on client Y

x_{23} = Time of PL B on client Z

x_{31}= Time of PL C on client X

x_{32}= Time of PL C on client Y

x_{33} = Time of PL C on client Z

Objective Function Min $Z = 10\ x_{11} + 16\ x_{12} + 32\ x_{13} + 14\ x_{21} + 22\ x_{22} + 40\ x_{23} + 22\ x_{31} + 24\ x_{32} + 34\ x_{33}$

Subject to constraints:

Each Project Leader must be assigned to one client

$x_{11} + x_{12} + x_{13} = 1$

$x_{21} + x_{22} + x_{23} = 1$

$x_{31} + x_{32} + x_{33} = 1$

Each client must be assigned to project leader

$x_{11} + x_{21} + x_{31} = 1$

$x_{12} + x_{22} + x_{32} = 1$

$x_{13} + x_{23} + x_{33} = 1$

3. Non-negativity Constraints : $x_{11}, x_{12}, x_{13} \ldots\ldots x_{32}, x_{33} \geq 0$